## Stochastic modeling of a serial killer

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We analyze the time pattern of the activity of a serial killer, who during twelve years had murdered 53 people. The plot of the cumulative number of murders as a function of time is of "Devil's staircase" type. The distribution of the intervals between murders (step length) follows a power law with the exponent of 1.4. We propose a model according to which the serial killer commits murders when neuronal excitation in his brain exceeds certain threshold. We model this neural activity as a branching process, which in turn is approximated by a random walk. As the distribution of the random walk return times is a power law with the exponent 1.5, the distribution of the inter-murder intervals is thus explained. We confirm analytical results by numerical simulation.

Figure 1 shows a time-plot of the cumulative number of murders committed by Andrei Chikatilo [1] during his twelve-year long career. It is highly irregular with long time intervals without murder interrupted by jumps, when he murdered many people during a short period. Such a curve is known in mathematics as a "Devil's staircase" [2]. We can characterize the staircase by the distributions of step lengths. Figure 2 shows such distributions for the staircase of Figure 1. The plots look linear in log-log coordinates, which suggest that the distributions follow a power law. The exponent of the power law of the probability density distribution (in the region of more than 16 days) is 1.4.

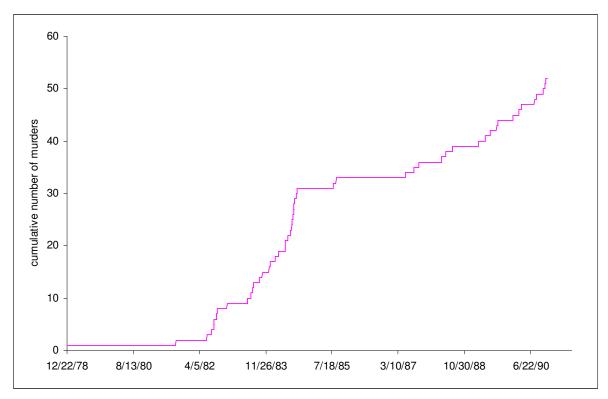
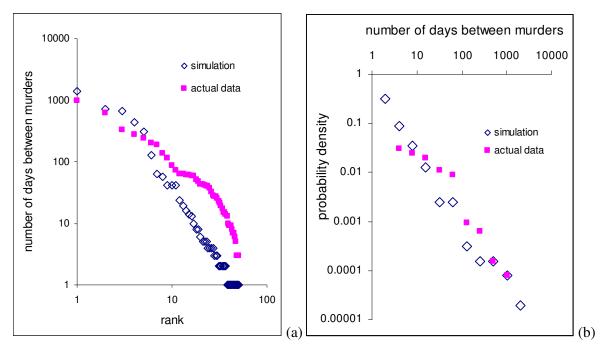


Figure 1. Chikatilo's staircase shows how the total number of his murders grew with time. The time span begins with his first murder on 12/22/1978 and ends with his arrest on 10/20/1990. The shortest interval between murders was three days and the longest – 986 days.



**Figure 2**. Distribution of step length (intervals between murders) in Zipfian (a) and probability density (b) representations.

Recently Osorio et al [3] reported a similar power-law distribution (with the exponent 1.5) of the intervals between epileptic seizures. We proposed a stochastic neural network model [4], which explained that finding. Here we apply a similar model to explain the distribution of intervals between murders. It may seem unreasonable to use the same model to describe an epileptic and a serial killer. However, Lombroso [5] long ago pointed out a link between epilepsy and criminality. A link between epilepsy and psychosis had been also established [6]. Thus, one may speculate that similar processes in the brain may lead to both epileptic seizures and serial killings.

We make a hypothesis that, similar to epileptic seizures, the psychotic affects, causing a serial killer to commit murder, arise from simultaneous firing of large number of neurons in the brain. Our neural net model for epileptics [4] and serial killers is as follows. After a neuron has fired, it cannot fire again for a time interval known as refractory period. Therefore, the minimum interval between the two subsequent firings of a neuron is the sum of spike duration and refractory period. This interval is few milliseconds and we will use it as our time unit. Consider one particular firing neuron. Its axon connects to synapses of thousands of other neurons. Some of them are almost ready to fire: their membrane potential is close to the firing threshold and the impulse from our neuron will be sufficient to surpass this threshold. These neurons will be firing next time step and they are can be called "children" of our neuron in the language of the theory of branching processes [7]. There are thousands of neurons that can be induced to fire by our neuron and the probability for each of them to be induced is small. Thus, the number of induced firings is Poisson distributed. If at given time step N neurons are firing then the number of neurons firing the next time step will come from a Poisson distribution with mean  $\lambda N$ . In addition to induced firings, some neurons will fire spontaneously. We assume that the number of spontaneously firing neurons at each time step comes from a Poisson distribution with mean p. The change in the number of firing neurons is

$$\Delta N = (\lambda - 1)N + p + \sqrt{N}z \tag{1}$$

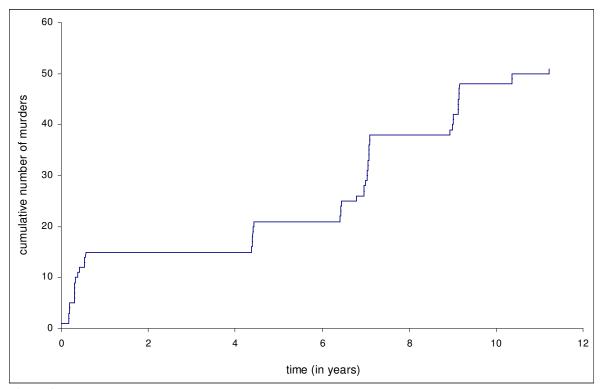
where z is a normally distributed random number with zero mean and unit variance. The number of firing neurons, N, performs a random walk, with the size of the step proportional to  $\sqrt{N}$ . We can simplify Eq.(1) by changing variable from N to  $x = \sqrt{N}$ . We get [4]:

$$\Delta x = \frac{(\lambda - 1)}{2} x - \left(\frac{1}{8} - \frac{p}{2}\right) \times \frac{1}{x} + \frac{1}{2} z$$
(2)

In the limit of large x, we can neglect the term, inversely proportional to x. When  $\lambda$  is very close to unity the first term can also be neglected. Equation (2) reduces to  $\Delta x = 1/2z$  which means that  $\sqrt{N}$  performs a simple random walk. A well know result in random walk theory is that the distribution of first return times follows a power law with an exponent of 3/2. We assume that the killer commits murder when the number of firing neurons reaches certain threshold. Then the distribution of first returns into murder zone (inter-murder intervals) is the same as the distribution of random walk's return times.

The model needs to be a bit more complex. We cannot expect that the killer commits murder right at the moment when neural excitation reaches a certain threshold. He needs time to plan and prepare his crime. So we assume that he commits murder after the neural excitation was over threshold for certain period d. Another assumption that we make is that a murder exercises a sedative effect on the killer, causing neural excitation to fall below the threshold. If we do not make this last assumption, the neural excitation will be in the murder zone for half of the time.

We made numerical simulations of the above model. We set  $\lambda = 1$  which corresponds to the critical branching process in neuron firing. This selection is not arbitrary since some experiments [8] suggest that neural circuits operate in critical regime. There are also theoretical reasons to believe that the brain functions in a critical state [9]. The system was simulated for  $2 \times 10^{11}$  time steps. Remember that time step is the sum of firing duration and refractory period.



**Figure 3.** Results of numerical simulation of the stochastic serial killer model. The distribution of step length is shown in Figure 2.

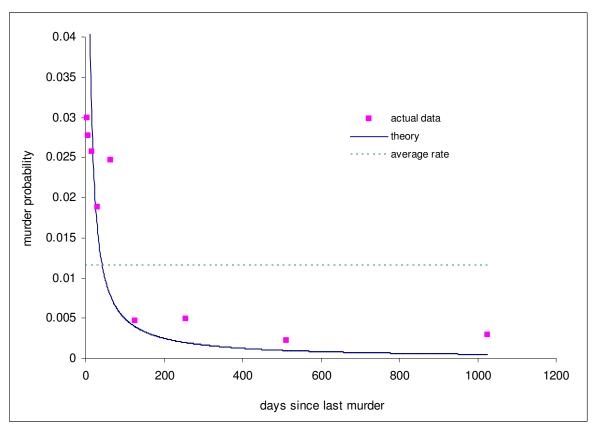
A reasonable estimate for this is two milliseconds. Thus, our simulation run corresponds to about twelve years. The rate of spontaneous firing was set at p = 0.1. The intensity threshold was set at  $10^9$  firing neurons. The time threshold, *d*, was set at 24 hours. Figures 2-3 show the results of these simulations. They decently agree with the experimental data.

The major disagreement is probably that the actual minimum number of days between murders is three, while the simulation produces a dozen inter-murder periods of one day. One could enhance the model by introducing a murder success rate. That is with certain probability everything goes well for the killer and he is able to commit the murder as he planned. If not, he repeats his attempt the next day. And so on. One could surely obtain a better agreement with experimental data, but this would be achieved by the price of introducing an extra parameter into the model.

An interesting question to ask is how the probability to commit a new murder depends on the time passed since the last murder. Suppose the killer committed his last murder n days ago. From the random-walk approximation, we immediately get that the probability to commit a murder

today is equal to  $\frac{1}{n^{3/2}} / \int_{n}^{\infty} \frac{dm}{m^{3/2}} = \frac{1}{2n}$ . Figure 4 shows this curve together with the actual data.

There is at least qualitative agreement between theory and observation. In particular, the probability of a new murder is significantly higher than the average murder rate immediately after murder and is significantly lower than the average murder rate when long time has passed since the last murder.



**Figure 4.** Daily murder probability as a function of the number of days passed since the last murder. The actual murder probability on *n*th day after previous murder is computed the following way. First, we divide the number of murders, which happened exactly on *n*th day by the total number of murders that happened on *n*th or later day. Afterward we average this over corresponding bins. The average murder rate is the total number of murders committed by Chikatilo divided by the length of the period during which he committed those murders.

## References

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